Matematika Diskrit P1–P7 – Cheat Sheet

TRPL Politeknik Negeri Batam Nama: _____ NIM: _____

Quick Reference

Himpunan. $|A|$ kardinalitas; $|P(A)|=2^{|A|}$; $|A\times B|=|A|\cdot|B|$; $|A\cup B|=|A|+|B|-|A\cap B|$; $|A\cup B\cup C|=|A|+|B|+|C|-|A\cap B|-|A\cap C|-|B\cap C|+|A\cap B\cap C|$; $\mu_A\in[0,1]$.

Matriks. $\det\left(\begin{smallmatrix}a&b\\c&d\end{smallmatrix}\right)=ad-bc$; $\left(\begin{smallmatrix}a&b\\c&d\end{smallmatrix}\right)^{-1}=\frac{1}{ad-bc}\left(\begin{smallmatrix}d&-b\\-c&a\end{smallmatrix}\right)$; $\det_{3\times3}=a(ei-fh)-b(di-fg)+c(dh-eg)$; $AX=B\Rightarrow X=A^{-1}B$.

Relasi. $R\subseteq A\times B$; $R^{-1}=\{(b,a)\mid(a,b)\in R\}$; $S\circ R=\{(a,c)\mid\exists b:(a,b)\in R\wedge(b,c)\in S\}$.

Fungsi. Injektif $f(a_1)=f(a_2)\Rightarrow a_1=a_2$; Surjektif $\forall b,\exists a:f(a)=b$; Bijektif $\Rightarrow f^{-1}$ ada; $(g\circ f)(x)=g(f(x))$.

Logika. $\neg(A\wedge B)\equiv\neg A\vee\neg B$; $\neg(A\vee B)\equiv\neg A\wedge\neg B$; $A\to B\equiv\neg A\vee B$; $A\leftrightarrow B\equiv(A\to B)\wedge(B\to A)$.

1. Himpunan

Definisi. Kumpulan objek berbeda terdefinisi jelas. $\{1,2,3\}$. Tidak boleh duplikat. $[1,2,2,3]=\{1,2,3\}$. *(Rosen Bab 1, Munir Bab 1)*

Notasi. $x\in A$; $x\notin A$; $|A|$; $\varnothing$; $A\subseteq B$; $A=B$; $A\sim B$ ekiv.

Prog. JS `Set`, SQL `DISTINCT`:

```
let s = new Set([1,2,2,3]);  // {1,2,3}
s.has(2);  // true;  s.size;  // 3
SELECT DISTINCT id FROM log;
```

4 Cara nyatakan. Enumerasi $\{101,102\}$; simbol baku $\mathbb{N},\mathbb{Z},\mathbb{Q},\mathbb{R},\mathbb{C}$; notasi pembentuk $\{x\mid100\le x\le115\}$; Venn.

Power set. $P(A)$ semua subset. $|P(A)|=2^{|A|}$. Contoh $A=\{a,b\}$: $P=\{\varnothing,\{a\},\{b\},\{a,b\}\}$.

6 Operasi. $A=\{1,2,3,4\}$, $B=\{3,4,5,6\}$: irisan $A\cap B=\{3,4\}$; gabungan $A\cup B=\{1,..,6\}$; selisih $A-B=\{1,2\}$; komplemen $\overline{A}=U-A$; beda simetris $A\oplus B=\{1,2,5,6\}$; kartesian $|A\times B|=|A|\cdot|B|$.

SQL analog. $\cap$=INNER JOIN; $\cup$=UNION; $-$=EXCEPT; $\times$=CROSS JOIN. $A\times B\neq B\times A$.

```
const A=new Set([1,2,3,4]), B=new Set([3,4,5,6]);
const i=[...A].filter(x=>B.has(x));   // [3,4]
const g=new Set([...A,...B]);
const s=[...A].filter(x=>!B.has(x)); // [1,2]
```

Inklusi-Eksklusi. $|A\cup B|=|A|+|B|-|A\cap B|$. Tiga: tanda $+-+$.

Contoh: 10 kopi, 8 teh, 3 keduanya $\to$ 15. 120 mhs: 60 MTK, 50 Fis, 40 Kim, 25 MTK$\cap$Fis, 20 MTK$\cap$Kim, 15 Fis$\cap$Kim, 10 ketiga $\to$ 100.

Fuzzy. $\mu\in[0,1]$. Usia 40: μ_{MUDA}=0,25, μ_{PAROBAYA}=0,5. Rekomendasi (0,7), spam (0,75), kualitas kode.

2. Matriks

Definisi. Ordo $m\times n$, elemen a_{ij}. Jenis: persegi, vektor, nol, identitas I_n (diag 1), transpos A^T. *(Anton-Rorres Bab 1, Strang Bab 1)*

Prog. Array 2D `M[i][j]`, numpy.

Operasi. Tambah/kurang: ordo sama. Skalar $c_{ij}=ka_{ij}$. **Kali matriks**: syarat kolom A=baris B, $c_{ij}=\sum_k a_{ik}b_{kj}$.

Contoh $A=\left(\begin{smallmatrix}1&2&3\\4&5&6\end{smallmatrix}\right)$, $B=\left(\begin{smallmatrix}7&8\\9&10\\11&12\end{smallmatrix}\right)$: c_{11}=58, c_{12}=64, c_{21}=139, c_{22}=154. $AB=\left(\begin{smallmatrix}58&64\\139&154\end{smallmatrix}\right)$. $AB\neq BA$.

```
function mul(A,B){let C=[];
 for(let i=0;i<A.length;i++){C[i]=[];
  for(let j=0;j<B[0].length;j++){let s=0;
   for(let k=0;k<B.length;k++)s+=A[i][k]*B[k][j];
   C[i][j]=s;}}return C;}
```

Det 2x2. $ad-bc$. Contoh $\left(\begin{smallmatrix}4&6\\3&8\end{smallmatrix}\right)\to14$.

Det 3x3 (Sarrus). $a(ei-fh)-b(di-fg)+c(dh-eg)$. Contoh $\left(\begin{smallmatrix}2&3&1\\1&2&1\\3&1&2\end{smallmatrix}\right)\to4$.

Sifat. det $=0$ singular (tak ada invers).

Invers 2x2. $\frac{1}{ad-bc}\left(\begin{smallmatrix}d&-b\\-c&a\end{smallmatrix}\right)$. Contoh $\left(\begin{smallmatrix}4&7\\2&6\end{smallmatrix}\right)\to\frac{1}{10}\left(\begin{smallmatrix}6&-7\\-2&4\end{smallmatrix}\right)$.

Invers 3x3. $\frac{1}{\det}$adj. Langkah: minor, kofaktor $(-1)^{i+j}$, adjugate transpose, bagi det.

Kesalahan. $A^{-1}\neq1/A$.

SPL. $AX=B\Rightarrow X=A^{-1}B$. Contoh $3x+2y=10, 5x+4y=20$: $X=\left(\begin{smallmatrix}0\\5\end{smallmatrix}\right)$. Contoh 2 $2x+y=8, x+3y=9$: $X=\left(\begin{smallmatrix}3\\2\end{smallmatrix}\right)$.

```
X = np.linalg.solve(A, B)
```

3. Relasi

Definisi. $R\subseteq A\times B$, $A\times B=\{(a,b)\mid a\in A,b\in B\}$. *(Rosen Bab 9, Munir Bab 4)*

3 Representasi. Diagram panah, tabel, matriks biner.

4 Sifat. - **Refleksif:** $(a,a)\in R$. Matriks: diag 1. - **Simetris:** $(a,b)\Rightarrow(b,a)$. Matriks: $M=M^T$. - **Antisimetris:** $(a,b),(b,a)\Rightarrow a=b$. Matriks: luar diag tidak ada pasangan. - **Transitif:** $(a,b),(b,c)\Rightarrow(a,c)$. Rantai punya jalan pintas.

Contoh $R=\{(1,1),(2,2),(2,3),(3,2),(4,2),(4,4)\}$: Ref ya, Sim tidak, Antisim tidak, Trans tidak.

Prog. Tabel relasional:

```
CREATE TABLE krs(mahasiswa VARCHAR(10),
 matakuliah VARCHAR(10));
```

Invers. $R^{-1}=\{(b,a)\mid(a,b)\in R\}$. Matriks = M^T. Invers relasi selalu ada.

Komposisi. $S\circ R$ baca "S setelah R". $(a,c)\in S\circ R$ jika $\exists b:(a,b)\in R,(b,c)\in S$.

Contoh $R=\{(1,2),(2,3),(3,4)\}$, $S=\{(2,a),(3,b),(4,c)\}\to\{(1,a),(2,b),(3,c)\}$.

```
SELECT r.mahasiswa, s.dosen FROM krs r
JOIN dosen s ON r.matakuliah=s.matakuliah;
```

4. Fungsi

Definisi. Setiap input tepat 1 output. Domain, kodomain, range. *(Rosen Bab 2, Munir Bab 5)*

3 Sifat. - **Injektif:** $f(a_1)=f(a_2)\Rightarrow a_1=a_2$. Syarat $|A|\le|B|$. - **Surjektif:** $\forall b,\exists a:f(a)=b$. Syarat $|A|\ge|B|$. - **Bijektif:** keduanya. Hanya ini punya invers.

Contoh $f=\{(1,a),(2,b),(3,c),(4,c)\}$: Inj tidak, Sur ya.

Invers. Syarat bijektif. Langkah: $y=f(x)$, cari x, tukar, uji.

Contoh $f=2x+3\to f^{-1}=(x-3)/2$. Uji $f(5)=13, f^{-1}(13)=5$.

Tidak punya. $f=x^2$: $f(3)=f(-3)$. Batasi $x\ge0$, $f^{-1}=\sqrt{x}$.

Kesalahan. $f^{-1}\neq1/f$.

Komposisi. $(g\circ f)(x)=g(f(x))$. Tidak komutatif. Contoh $f=3x+1, g=x^2$: $(g\circ f)(2)=49$, $(f\circ g)(2)=13$.

Khusus. $\lfloor3{,}7\rfloor=3$; $\lceil3{,}2\rceil=4$; $17\bmod5=2$; $-7\bmod3=2$ Py, -1 JS; $0!=1, 5!=120$.

Rekursif. Basis + rekurens:

```
function fakt(n){
 if(n===0)return 1;    // basis
 return n*fakt(n-1);   // rekurens
}
```

Tanpa basis $\to$ stack overflow.

5. Logika

Proposisi. B/S tidak keduanya. Bukan: pertanyaan, perintah, variabel bebas. *(Rosen Bab 1, O'Regan Bab 10)*

5 Operator. $\neg,\wedge,\vee,\to,\leftrightarrow$.

A	B	$\neg A$	$A\wedge B$	$A\vee B$	A	B	$A\to B$	$A\leftrightarrow B$
B	B	S	B	B	B	B	B	B
B	S	S	S	B	B	S	S	S
S	B	B	S	B	S	B	B	S
S	S	B	S	S	S	S	B	B

Implikasi. Salah hanya jika A B, B S. A salah $\to$ implikasi B (vacuous).

JS analog. $\neg$:`!A`; $\wedge$:`A&&B`; $\vee$:`A||B`; $\to$:`!A||B`; $\leftrightarrow$:`A===B`.

Klasifikasi. Tautologi selalu B ($A\vee\neg A$). Kontradiksi selalu S ($A\wedge\neg A$). Kontingensi campuran.

Hukum. - Komut, Asos, Distrib - Ident $A\wedge B\equiv A$ - Idem $A\wedge A\equiv A$ - Dominasi $A\vee B\equiv B$ - Neg ganda $\neg\neg A\equiv A$ - Excl middle $A\vee\neg A\equiv B$ - **De Morgan** $\neg(A\wedge B)\equiv\neg A\vee\neg B$; $\neg(A\vee B)\equiv\neg A\wedge\neg B$ - **Implikasi** $A\to B\equiv\neg A\vee B$

JS De Morgan. `!(a&&b)`$\equiv$`!a||!b`; `!(a||b)`$\equiv$`!a&&!b`.

Inferensi. Ponens $A\to B, A\Rightarrow B$. Tollens $A\to B,\neg B\Rightarrow\neg A$. Silogisme $A\to B, B\to C\Rightarrow A\to C$.

Contoh: hujan$\to$basah, tidak basah $\Rightarrow$ tidak hujan.

Sederhana 1. $\neg(\neg A\vee B)\vee(A\wedge B)\equiv(A\wedge\neg B)\vee(A\wedge B)\equiv A\wedge(\neg B\vee B)\equiv A$.

Sederhana 2. $\neg(A\wedge\neg B)\wedge(A\vee B)\equiv(\neg A\vee B)\wedge(A\vee B)\equiv(\neg A\wedge A)\vee B\equiv B$.

Studi kasus. $(L\wedge A\wedge O)\wedge\neg B$. 16 baris, hanya 1 B.

```
if(isLogin && isAdmin && otpValid && !blocked)
  showDashboard();
```

6. Soal Campuran

S1. $A=\{1,2,3,4,5\}, B=\{4,5,6,7\}, U=\{1..10\}$. $\cap=\{4,5\}, \cup=\{1..7\}, -=\{1,2,3\}, \bar{A}=\{6$

S2. Pramuka 30, Basket 25, kdg 15: $\cup$=40, tidak $50-40=10$.

S3. $A=\left(\begin{smallmatrix}2&3\\1&4\end{smallmatrix}\right), B=\left(\begin{smallmatrix}1&0\\2&5\end{smallmatrix}\right)$. $AB=\left(\begin{smallmatrix}8&15\\9&20\end{smallmatrix}\right)$, det $=5$, $A^{-1}=\left(\begin{smallmatrix}0{,}8&-0{,}6\\-0{,}2&0{,}4\end{smallmatrix}\right)$.

S4. $2x+y=8, x+3y=9$: $x=3, y=2$.

S5. $R=\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$: Ref ya, Sim ya, Antisim tidak, Trans ya.

S6. $R=\{(1,2),(2,3),(3,4)\}, S=\{(2,a),(3,b),(4,c)\} \to \{(1,a),(2,b),(3,c)\}$.

S7. $f=3x-2 \to f^{-1}=(x+2)/3$.

S8. $f=x+2, g=x^2$: $(g \circ f)(3)=25, (f \circ g)(3)=11$.

S9. $\neg(A \wedge \neg B) \wedge (A \vee B) \equiv B$.

S10. Fuzzy: $0{,}9 > 0{,}7 > 0{,}5 \to A > C > B$.

7. Kesalahan + Tips

Kesalahan. Larik mentah→Set/DISTINCT; lupa irisan→kurangi $|A \cap B|$; $A \times B = B \times A$? tidak; beda ordo→cek dulu; kali per elemen→baris×kolom; invers tanpa cek det; tidak setangkup≠antisimetris; invers relasi pakai M^{-1}? balik pasangan; $S \circ R$ kiri→kanan? baca dari kanan; $f^{-1}=1/f$? tidak; rekursi tanpa basis→overflow; tabel kurang→2^n baris; lupa negasi blokir; De Morgan salah→$\wedge \leftrightarrow \vee$.

Tips. "total"→inklusi; "hanya"→selisih. Matriks: ordo→det→invers. $AB \neq BA$. Relasi: matriks biner, 4 sifat terpisah. Fungsi: bijektif dulu. Logika: De Morgan+impl untuk sederhana; tabel untuk validitas. Waktu: hitung dulu, bukti belakangan. Macet: tulis diketahui+rumus+langkah.

Rujukan. Rosen 2019; Munir 2012; Epp 2010; Anton-Rorres 2014; Strang 2016; O'Regan 2023; Silberschatz 2019.